

5.5

Inequalities in One Triangle

What you should learn

GOAL 1 Use triangle measurements to decide which side is longest or which angle is largest, as applied in **Example 2**.

GOAL 2 Use the Triangle Inequality.

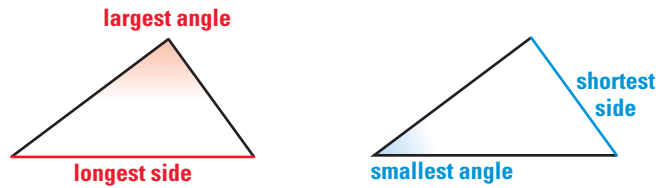
Why you should learn it

▼ To solve **real-life** problems, such as describing the motion of a crane as it clears the sediment from the mouth of a river in **Exs. 29–31**.



GOAL 1 COMPARING MEASUREMENTS OF A TRIANGLE

In Activity 5.5, you may have discovered a relationship between the positions of the longest and shortest sides of a triangle and the positions of its angles.

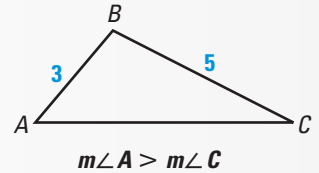


The diagrams illustrate the results stated in the theorems below.

THEOREMS

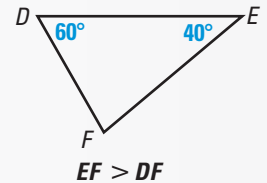
THEOREM 5.10

If one side of a triangle is longer than another side, then the angle opposite the longer side is larger than the angle opposite the shorter side.



THEOREM 5.11

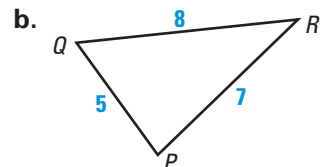
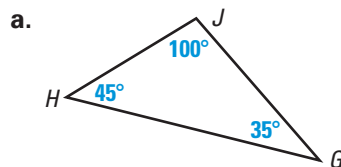
If one angle of a triangle is larger than another angle, then the side opposite the larger angle is longer than the side opposite the smaller angle.



You can write the measurements of a triangle in order from least to greatest.

EXAMPLE 1 Writing Measurements in Order from Least to Greatest

Write the measurements of the triangles in order from least to greatest.



SOLUTION

a. $m\angle G < m\angle H < m\angle J$
 $JH < JG < GH$

b. $QP < PR < QR$
 $m\angle R < m\angle Q < m\angle P$

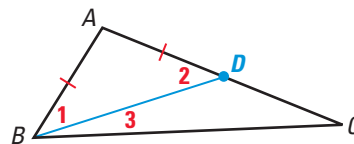


Theorem 5.11 will be proved in Lesson 5.6, using a technique called *indirect proof*. Theorem 5.10 can be proved using the diagram shown below.

GIVEN $\triangleright AC > AB$

PROVE $\triangleright m\angle ABC > m\angle C$

Paragraph Proof Use the Ruler Postulate to locate a point D on \overline{AC} such that $DA = BA$. Then draw the segment \overline{BD} . In the isosceles triangle $\triangle ABD$, $\angle 1 \cong \angle 2$. Because $m\angle ABC = m\angle 1 + m\angle 3$, it follows that $m\angle ABC > m\angle 1$. Substituting $m\angle 2$ for $m\angle 1$ produces $m\angle ABC > m\angle 2$. Because $m\angle 2 = m\angle 3 + m\angle C$, $m\angle 2 > m\angle C$. Finally, because $m\angle ABC > m\angle 2$ and $m\angle 2 > m\angle C$, you can conclude that $m\angle ABC > m\angle C$.



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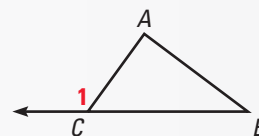
The proof of Theorem 5.10 above uses the fact that $\angle 2$ is an exterior angle for $\triangle BDC$, so its measure is the sum of the measures of the two nonadjacent interior angles. Then $m\angle 2$ must be greater than the measure of either nonadjacent interior angle. This result is stated below as Theorem 5.12.

THEOREM

THEOREM 5.12 Exterior Angle Inequality

The measure of an exterior angle of a triangle is greater than the measure of either of the two nonadjacent interior angles.

$$m\angle 1 > m\angle A \text{ and } m\angle 1 > m\angle B$$



You can use Theorem 5.10 to determine possible angle measures in a chair or other real-life object.

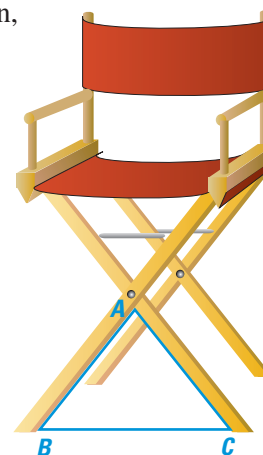
EXAMPLE 2 Using Theorem 5.10



DIRECTOR'S CHAIR In the director's chair shown, $\overline{AB} \cong \overline{AC}$ and $BC > AB$. What can you conclude about the angles in $\triangle ABC$?

SOLUTION

Because $\overline{AB} \cong \overline{AC}$, $\triangle ABC$ is isosceles, so $\angle B \cong \angle C$. Therefore, $m\angle B = m\angle C$. Because $BC > AB$, $m\angle A > m\angle C$ by Theorem 5.10. By substitution, $m\angle A > m\angle B$. In addition, you can conclude that $m\angle A > 60^\circ$, $m\angle B < 60^\circ$, and $m\angle C < 60^\circ$.



GOAL 2 USING THE TRIANGLE INEQUALITY

Not every group of three segments can be used to form a triangle. The lengths of the segments must fit a certain relationship.

EXAMPLE 3 Constructing a Triangle

Construct a triangle with the given group of side lengths, if possible.

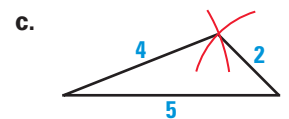
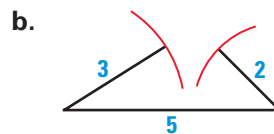
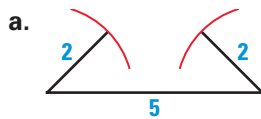
a. 2 cm, 2 cm, 5 cm

b. 3 cm, 2 cm, 5 cm

c. 4 cm, 2 cm, 5 cm

SOLUTION

Try drawing triangles with the given side lengths. Only group (c) is possible. The sum of the first and second lengths must be greater than the third length.



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The result of Example 3 is summarized as Theorem 5.13. Exercise 34 asks you to write a proof of this theorem.

THEOREM

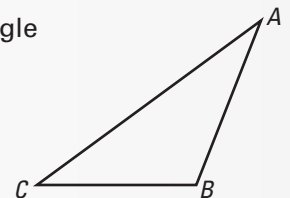
THEOREM 5.13 Triangle Inequality

The sum of the lengths of any two sides of a triangle is greater than the length of the third side.

$$AB + BC > AC$$

$$AC + BC > AB$$

$$AB + AC > BC$$



EXAMPLE 4 Finding Possible Side Lengths

A triangle has one side of 10 centimeters and another of 14 centimeters. Describe the possible lengths of the third side.

SOLUTION

Let x represent the length of the third side. Using the Triangle Inequality, you can write and solve inequalities.

$$x + 10 > 14$$

$$x > 4$$

$$10 + 14 > x$$

$$24 > x$$

▶ So, the length of the third side must be greater than 4 centimeters and less than 24 centimeters.

STUDENT HELP

Skills Review

For help with solving inequalities, see p. 791.

GUIDED PRACTICE

Vocabulary Check ✓

1. $\triangle ABC$ has side lengths of 1 inch, $1\frac{7}{8}$ inches, and $2\frac{1}{8}$ inches and angle measures of 90° , 28° , and 62° . Which side is *opposite* each angle?

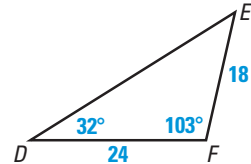
Concept Check ✓

2. Is it possible to draw a triangle with side lengths of 5 inches, 2 inches, and 8 inches? Explain why or why not.

Skill Check ✓

In Exercises 3 and 4, use the figure shown at the right.

3. Name the smallest and largest angles of $\triangle DEF$.
4. Name the shortest and longest sides of $\triangle DEF$.



5. **GEOGRAPHY** Suppose you know the following information about distances between cities in the Philippine Islands:

Cadiz to Masbate: 99 miles

Cadiz to Guiuan: 165 miles

Describe the range of possible distances from Guiuan to Masbate.

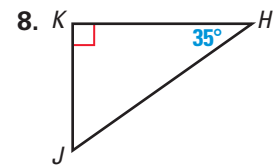
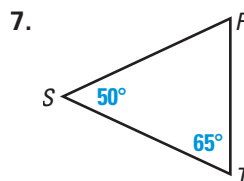
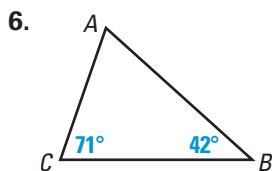


PRACTICE AND APPLICATIONS

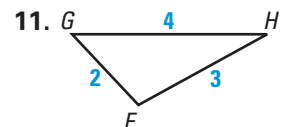
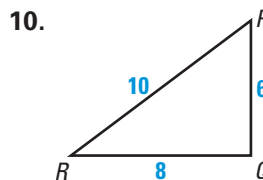
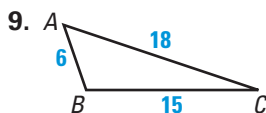
STUDENT HELP

Extra Practice to help you master skills is on p. 812.

COMPARING SIDE LENGTHS Name the shortest and longest sides of the triangle.



COMPARING ANGLE MEASURES Name the smallest and largest angles of the triangle.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 6–19

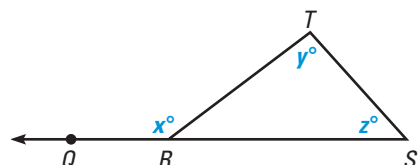
Example 2: Exs. 6–19

Example 3: Exs. 20–23

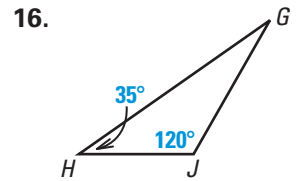
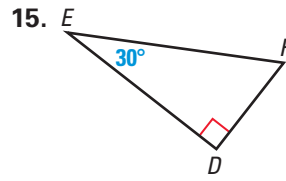
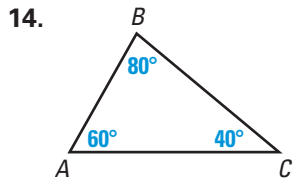
Example 4: Exs. 24, 25

USING ALGEBRA Use the diagram of $\triangle RST$ with exterior angle $\angle QRT$.

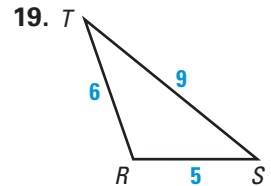
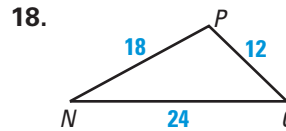
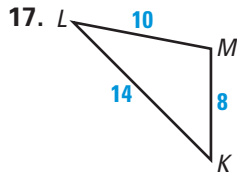
12. Write an equation about the angle measures labeled in the diagram.
13. Write two inequalities about the angle measures labeled in the diagram.



ORDERING SIDES List the sides in order from shortest to longest.



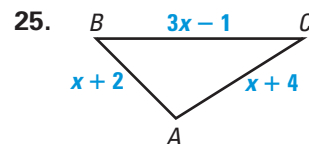
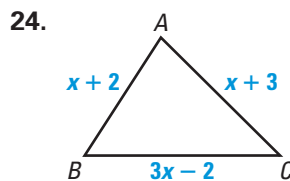
ORDERING ANGLES List the angles in order from smallest to largest.



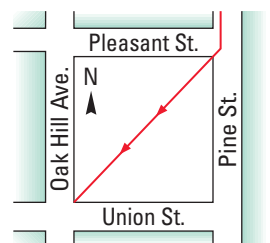
FORMING TRIANGLES In Exercises 20–23, you are given an 18 inch piece of wire. You want to bend the wire to form a triangle so that the length of each side is a whole number.

- Sketch four possible isosceles triangles and label each side length.
- Sketch a possible acute scalene triangle.
- Sketch a possible obtuse scalene triangle.
- List three combinations of segment lengths that will not produce triangles.

xy USING ALGEBRA In Exercises 24 and 25, solve the inequality $AB + AC > BC$.



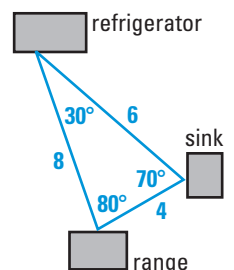
26. **TAKING A SHORTCUT** Look at the diagram shown. Suppose you are walking south on the sidewalk of Pine Street. When you reach Pleasant Street, you cut across the empty lot to go to the corner of Oak Hill Avenue and Union Street. Explain why this route is shorter than staying on the sidewalks.



KITCHEN TRIANGLE In Exercises 27 and 28, use the following information.

The term “kitchen triangle” refers to the imaginary triangle formed by three kitchen appliances: the refrigerator, the sink, and the range. The distances shown are measured in feet.

- What is wrong with the labels on the kitchen triangle?
- Can a kitchen triangle have the following side lengths: 9 feet, 3 feet, and 5 feet? Explain why or why not.



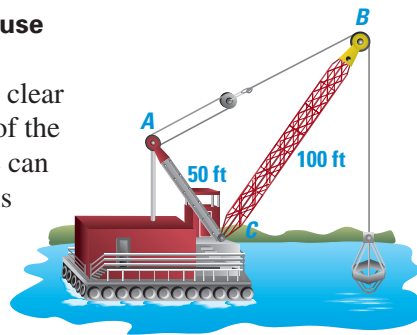
FOCUS ON APPLICATIONS

REAL LIFE WORK TRIANGLES

For ease of movement among appliances, the perimeter of an ideal kitchen work triangle should be less than 22 ft and more than 15 ft.

CHANNEL DREDGING In Exercises 29–31, use the figure shown and the given information.

The crane is used in dredging mouths of rivers to clear out the collected debris. By adjusting the length of the boom lines from A to B , the operator of the crane can raise and lower the boom. Suppose the mast \overline{AC} is 50 feet long and the boom \overline{BC} is 100 feet long.



29. Is the boom *raised* or *lowered* when the boom lines are shortened?
30. AB must be less than ? feet.
31. As the boom and shovel are raised or lowered, is $\angle ACB$ ever larger than $\angle BAC$? Explain.

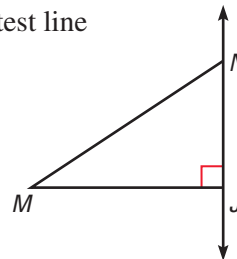
32. **LOGICAL REASONING** In Example 4 on page 297, only two inequalities were needed to solve the problem. Write the third inequality. Why is that inequality not helpful in determining the range of values of x ?

33. **PROOF** Prove that a perpendicular segment is the shortest line segment from a point to a line. Prove that \overline{MJ} is the shortest line segment from M to \overleftrightarrow{JN} .

GIVEN $\triangleright \overline{MJ} \perp \overleftrightarrow{JN}$

PROVE $\triangleright MN > MJ$

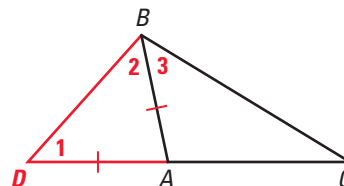
Plan for Proof Show that $m\angle MJN > m\angle MNJ$, so $MN > MJ$.



34. **DEVELOPING PROOF** Complete the proof of Theorem 5.13, the Triangle Inequality.

GIVEN $\triangleright \triangle ABC$

- PROVE** \triangleright (1) $AB + BC > AC$
 (2) $AC + BC > AB$
 (3) $AB + AC > BC$



Plan for Proof One side, say \overline{BC} , is longer than or is at least as long as each of the other sides. Then (1) and (2) are true. The proof for (3) is as follows.

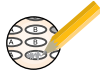
Statements	Reasons
1. $\triangle ABC$	1. Given
2. Extend \overline{AC} to D such that $\overline{AB} \cong \overline{AD}$.	2. <u>?</u>
3. $AD + AC =$ <u>?</u>	3. Segment Addition Postulate
4. $\angle 1 \cong \angle 2$	4. <u>?</u>
5. $m\angle DBC >$ <u>?</u>	5. Protractor Postulate
6. $m\angle DBC >$ $m\angle 1$	6. <u>?</u>
7. $DC >$ BC	7. <u>?</u>
8. <u>?</u> + <u>?</u> $>$ BC	8. Substitution property of equality
9. $AB + AC >$ <u>?</u>	9. Substitution property of equality

STUDENT HELP

INTERNET

HOMEWORK HELP
 Visit our Web site
www.mcdougallittell.com
 for help with proof.

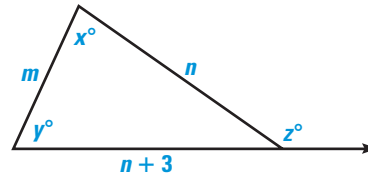
Test Preparation



QUANTITATIVE COMPARISON In Exercises 35–37, use the diagram to choose the statement that is true about the given quantities.

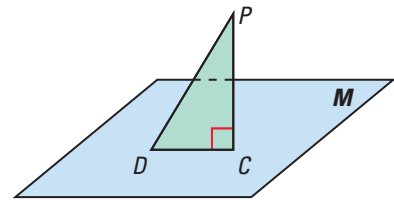
- (A) The quantity in column A is greater.
- (B) The quantity in column B is greater.
- (C) The two quantities are equal.
- (D) The relationship cannot be determined from the given information.

	Column A	Column B
35.	x	y
36.	x	z
37.	m	n



★ Challenge

38. **PROOF** Use the diagram shown to prove that a perpendicular segment is the shortest segment from a point to a plane.



GIVEN $\overline{PC} \perp$ plane M

PROVE $PD > PC$

EXTRA CHALLENGE

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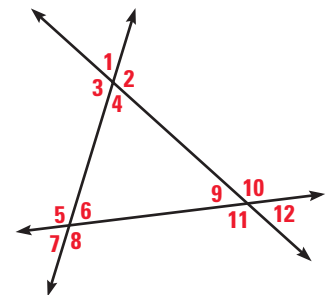
MIXED REVIEW

RECOGNIZING PROOFS In Exercises 39–41, look through your textbook to find an example of the type of proof. (Review Chapters 2–5 for 5.6)

- 39. two-column proof
- 40. paragraph proof
- 41. flow proof

ANGLE RELATIONSHIPS Complete each statement. (Review 3.1)

- 42. $\angle 5$ and $\underline{\quad ? \quad}$ are corresponding angles. So are $\angle 5$ and $\underline{\quad ? \quad}$.
- 43. $\angle 12$ and $\underline{\quad ? \quad}$ are vertical angles.
- 44. $\angle 6$ and $\underline{\quad ? \quad}$ are alternate interior angles. So are $\angle 6$ and $\underline{\quad ? \quad}$.
- 45. $\angle 7$ and $\underline{\quad ? \quad}$ are alternate exterior angles. So are $\angle 7$ and $\underline{\quad ? \quad}$.



xy USING ALGEBRA In Exercises 46–49, you are given the coordinates of the midpoints of the sides of a triangle. Find the coordinates of the vertices of the triangle. (Review 5.4)

- 46. $L(-2, 1)$, $M(2, 3)$, $N(3, -1)$
- 47. $L(-3, 5)$, $M(-2, 2)$, $N(-6, 0)$
- 48. $L(3, 6)$, $M(9, 5)$, $N(8, 1)$
- 49. $L(3, -2)$, $M(0, -4)$, $N(3, -6)$