

3.6

Parallel Lines in the Coordinate Plane

What you should learn

GOAL 1 Find slopes of lines and use slope to identify parallel lines in a coordinate plane.

GOAL 2 Write equations of parallel lines in a coordinate plane.

Why you should learn it

▼ To describe steepness in **real-life**, such as the cog railway in **Example 1** and the zip line in **Ex. 46**.



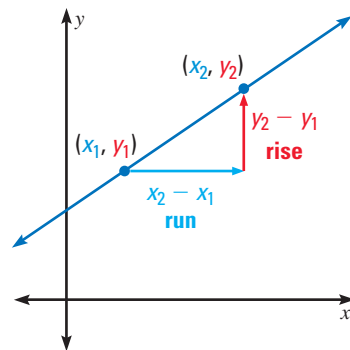
GOAL 1 SLOPE OF PARALLEL LINES

In algebra, you learned that the slope of a nonvertical line is the ratio of the vertical change (the rise) to the horizontal change (the run). If the line passes through the points (x_1, y_1) and (x_2, y_2) , then the slope is given by

$$\text{Slope} = \frac{\text{rise}}{\text{run}}$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

Slope is usually represented by the variable m .



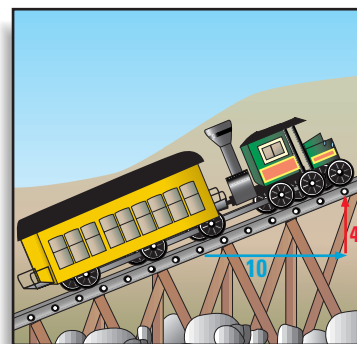
EXAMPLE 1 Finding the Slope of Train Tracks



COG RAILWAY A cog railway goes up the side of Mount Washington, the tallest mountain in New England. At the steepest section, the train goes up about 4 feet for each 10 feet it goes forward. What is the slope of this section?

SOLUTION

$$\text{slope} = \frac{\text{rise}}{\text{run}} = \frac{4 \text{ feet}}{10 \text{ feet}} = 0.4$$



EXAMPLE 2 Finding the Slope of a Line

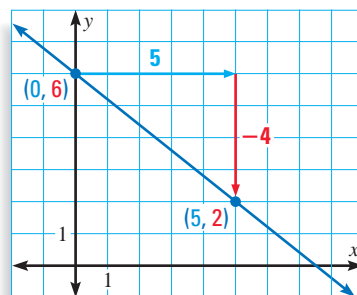
Find the slope of the line that passes through the points $(0, 6)$ and $(5, 2)$.

SOLUTION

Let $(x_1, y_1) = (0, 6)$ and $(x_2, y_2) = (5, 2)$.

$$\begin{aligned} m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{2 - 6}{5 - 0} \\ &= -\frac{4}{5} \end{aligned}$$

► The slope of the line is $-\frac{4}{5}$.

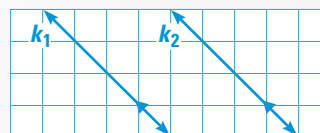


You can use the slopes of two lines to tell whether the lines are parallel.

POSTULATE

POSTULATE 17 *Slopes of Parallel Lines*

In a coordinate plane, two nonvertical lines are parallel if and only if they have the same slope. Any two vertical lines are parallel.



Lines k_1 and k_2 have the same slope.

STUDENT HELP

Study Tip

To find the slope in Example 3, you can either use the slope formula or you can count units on the graph.

EXAMPLE 3 *Deciding Whether Lines are Parallel*

Find the slope of each line. Is $j_1 \parallel j_2$?

SOLUTION

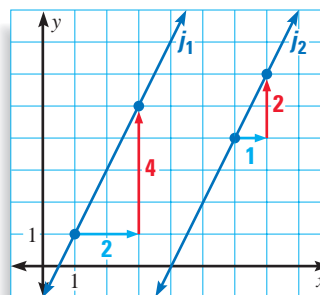
Line j_1 has a slope of

$$m_1 = \frac{4}{2} = 2$$

Line j_2 has a slope of

$$m_2 = \frac{2}{1} = 2$$

▶ Because the lines have the same slope, $j_1 \parallel j_2$.



EXAMPLE 4 *Identifying Parallel Lines*

Find the slope of each line. Which lines are parallel?

SOLUTION

Find the slope of k_1 . Line k_1 passes through $(0, 6)$ and $(2, 0)$.

$$m_1 = \frac{0 - 6}{2 - 0} = \frac{-6}{2} = -3$$

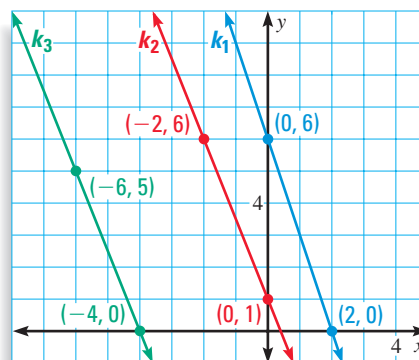
Find the slope of k_2 . Line k_2 passes through $(-2, 6)$ and $(0, 1)$.

$$m_2 = \frac{1 - 6}{0 - (-2)} = \frac{-5}{0 + 2} = -\frac{5}{2}$$

Find the slope of k_3 . Line k_3 passes through $(-6, 5)$ and $(-4, 0)$.

$$m_3 = \frac{0 - 5}{-4 - (-6)} = \frac{-5}{-4 + 6} = -\frac{5}{2}$$

▶ Compare the slopes. Because k_2 and k_3 have the same slope, they are parallel. Line k_1 has a different slope, so it is not parallel to either of the other lines.



GOAL 2 WRITING EQUATIONS OF PARALLEL LINES

In algebra, you learned that you can use the slope m of a nonvertical line to write an equation of the line in *slope-intercept form*.

$$y = mx + b$$

slope \swarrow m \nwarrow b y-intercept

The y-intercept is the y-coordinate of the point where the line crosses the y-axis.



EXAMPLE 5 Writing an Equation of a Line

Write an equation of the line through the point $(2, 3)$ that has a slope of 5.

SOLUTION

Solve for b . Use $(x, y) = (2, 3)$ and $m = 5$.

$$y = mx + b \quad \text{Slope-intercept form}$$

$$3 = 5(2) + b \quad \text{Substitute 2 for } x, 3 \text{ for } y, \text{ and 5 for } m.$$

$$3 = 10 + b \quad \text{Simplify.}$$

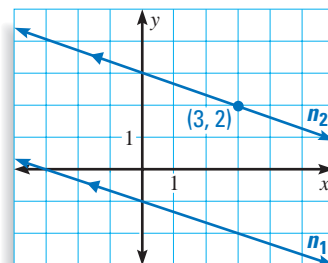
$$-7 = b \quad \text{Subtract.}$$

► Write an equation. Since $m = 5$ and $b = -7$, an equation of the line is $y = 5x - 7$.

EXAMPLE 6 Writing an Equation of a Parallel Line

Line n_1 has the equation $y = -\frac{1}{3}x - 1$.

Line n_2 is parallel to n_1 and passes through the point $(3, 2)$. Write an equation of n_2 .



SOLUTION

Find the slope.

The slope of n_1 is $-\frac{1}{3}$. Because parallel lines have the same slope, the slope of n_2 is also $-\frac{1}{3}$.

Solve for b . Use $(x, y) = (3, 2)$ and $m = -\frac{1}{3}$.

$$y = mx + b$$

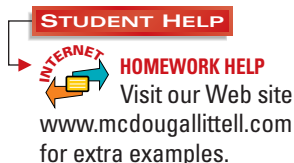
$$2 = -\frac{1}{3}(3) + b$$

$$2 = -1 + b$$

$$3 = b$$

Write an equation.

► Because $m = -\frac{1}{3}$ and $b = 3$, an equation of n_2 is $y = -\frac{1}{3}x + 3$.



GUIDED PRACTICE

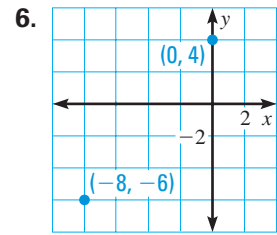
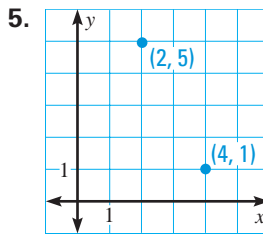
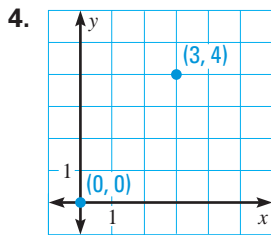
Vocabulary Check ✓

Concept Check ✓

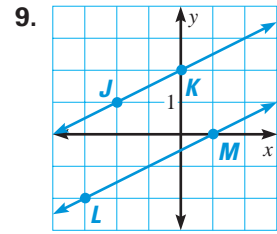
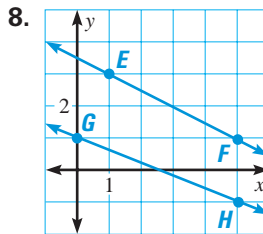
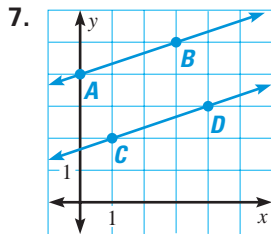
Skill Check ✓

1. What does *intercept* mean in the expression *slope-intercept form*?
2. The slope of line j is 2 and $j \parallel k$. What is the slope of line k ?
3. What is the slope of a horizontal line? What is the slope of a vertical line?

Find the slope of the line that passes through the labeled points.



Determine whether the two lines shown in the graph are parallel. If they are parallel, explain how you know.



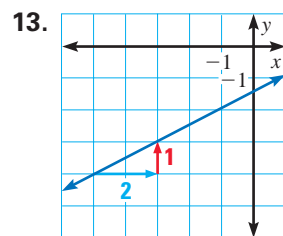
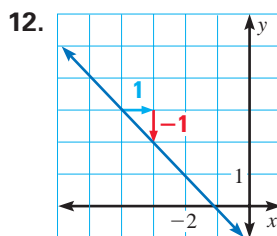
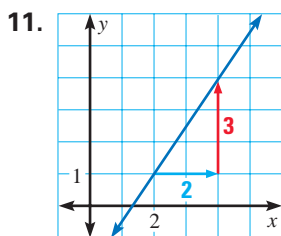
10. Write an equation of the line that passes through the point $(2, -3)$ and has a slope of -1 .

PRACTICE AND APPLICATIONS

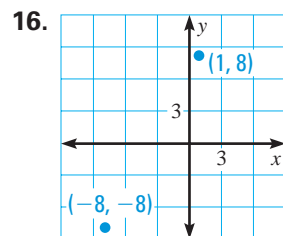
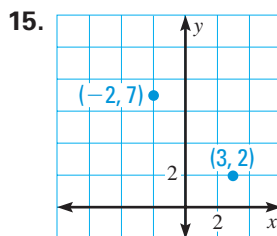
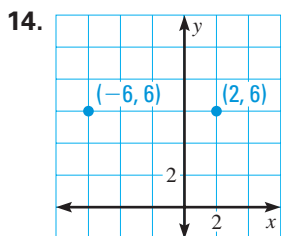
STUDENT HELP

Extra Practice to help you master skills is on p. 808.

CALCULATING SLOPE What is the slope of the line?



CALCULATING SLOPE Find the slope of the line that passes through the labeled points on the graph.



STUDENT HELP

HOMEWORK HELP

Example 1: Exs. 11–16, 23, 46, 49–52
Example 2: Exs. 11–16

STUDENT HELP

▶ HOMEWORK HELP

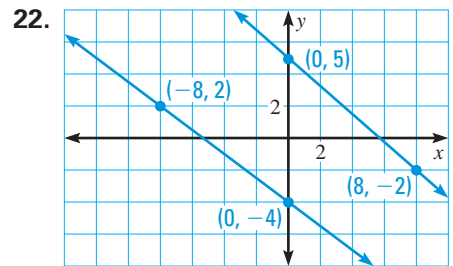
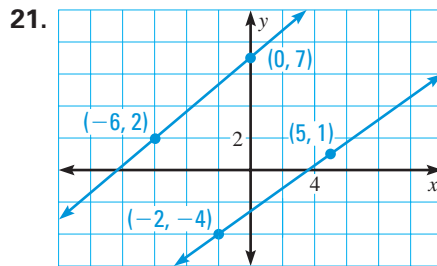
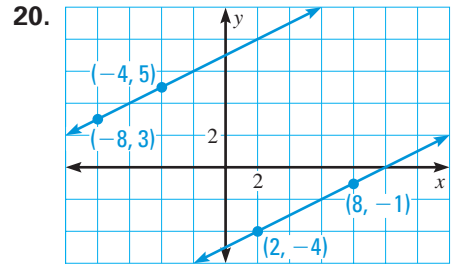
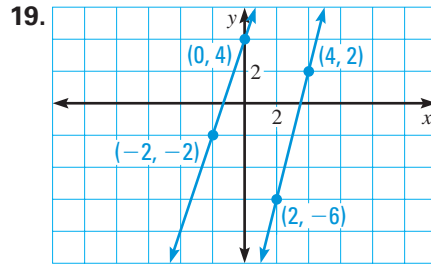
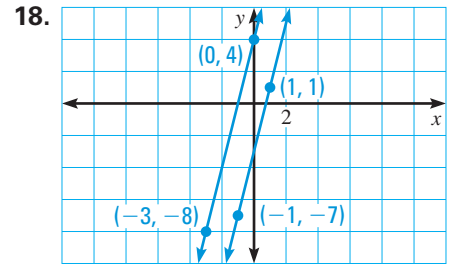
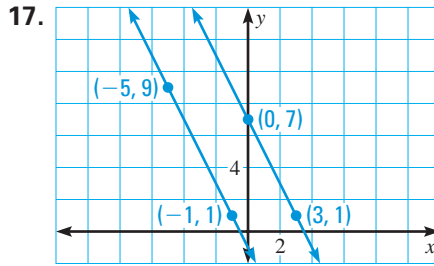
Example 3: Exs. 17–22

Example 4: Exs. 24–26,
47, 48

Example 5: Exs. 27–41

Example 6: Exs. 42–45

IDENTIFYING PARALLELS Find the slope of each line. Are the lines parallel?



23. **UNDERGROUND RAILROAD** The photo at the right shows a monument in Oberlin, Ohio, that is dedicated to the Underground Railroad. The slope of each of the rails is about $-\frac{3}{5}$ and the sculpture is about 12 feet long. What is the height of the ends of the rails? Explain how you found your answer.



FOCUS ON PEOPLE



UNDERGROUND RAILROAD is the name given to the network of people who helped some slaves to freedom. Harriet Tubman, a former slave, helped about 300 escape.

IDENTIFYING PARALLELS Find the slopes of \overleftrightarrow{AB} , \overleftrightarrow{CD} , and \overleftrightarrow{EF} . Which lines are parallel, if any?

- | | | |
|--|---|---|
| 24. $A(0, -6), B(4, -4)$
$C(0, 2), D(2, 3)$
$E(0, -4), F(1, -7)$ | 25. $A(2, 6), B(4, 7)$
$C(0, -1), D(6, 2)$
$E(4, -5), F(8, -2)$ | 26. $A(-4, 10), B(-8, 7)$
$C(-5, 7), D(-2, 4)$
$E(2, -3), F(6, -7)$ |
|--|---|---|

WRITING EQUATIONS Write an equation of the line.

- | | | |
|--|---|--|
| 27. slope = 3
y-intercept = 2 | 28. slope = $\frac{1}{3}$
y-intercept = -4 | 29. slope = $-\frac{2}{9}$
y-intercept = 0 |
| 30. slope = $\frac{1}{2}$
y-intercept = 6 | 31. slope = 0
y-intercept = -3 | 32. slope = $-\frac{2}{9}$
y-intercept = $-\frac{3}{5}$ |

WRITING EQUATIONS Write an equation of the line that has a y -intercept of 3 and is parallel to the line whose equation is given.

33. $y = -6x + 2$

34. $y = x - 8$

35. $y = -\frac{4}{3}x$

WRITING EQUATIONS Write an equation of the line that passes through the given point P and has the given slope.

36. $P(0, -6), m = -2$

37. $P(-3, 9), m = -1$

38. $P\left(\frac{3}{2}, 4\right), m = \frac{1}{2}$

39. $P(2, -4), m = 0$

40. $P(-7, -5), m = \frac{3}{4}$

41. $P(6, 1)$, undefined slope

xy USING ALGEBRA Write an equation of the line that passes through point P and is parallel to the line with the given equation.

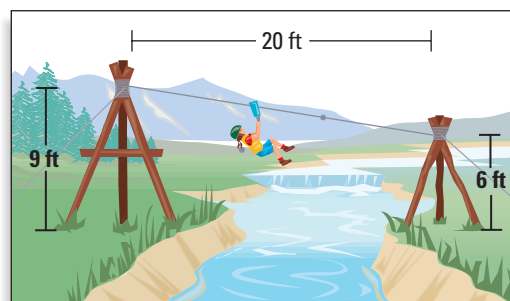
42. $P(-3, 6), y = -x - 5$

43. $P(1, -2), y = \frac{5}{4}x - 8$

44. $P(8, 7), y = 3$

45. **xy USING ALGEBRA** Write an equation of a line parallel to $y = \frac{1}{3}x - 16$.

46. **ZIP LINE** A zip line is a taut rope or cable that you can ride down on a pulley. The zip line at the right goes from a 9 foot tall tower to a 6 foot tall tower. The towers are 20 feet apart. What is the slope of the zip line?



COORDINATE GEOMETRY In Exercises 47 and 48, use the five points: $P(0, 0)$, $Q(1, 3)$, $R(4, 0)$, $S(8, 2)$, and $T(9, 5)$.

47. Plot and label the points. Connect every pair of points with a segment.
48. Which segments are parallel? How can you verify this?

CIVIL ENGINEERING In Exercises 49–52, use the following information. The slope of a road is called the road's *grade*. Grades are measured in percents. For example, if the slope of a road is $\frac{1}{20}$, the grade is 5%. A warning sign is needed before any hill that fits one of the following descriptions.

- 5% grade and more than 3000 feet long
- 6% grade and more than 2000 feet long
- 7% grade and more than 1000 feet long
- 8% grade and more than 750 feet long
- 9% grade and more than 500 feet long

► Source: U.S. Department of Transportation



What is the grade of the hill to the nearest percent? Is a sign needed?

- 49. The hill is 1400 feet long and drops 70 feet.
- 50. The hill is 2200 feet long and drops 140 feet.
- 51. The hill is 600 feet long and drops 55 feet.
- 52. The hill is 450 feet long and drops 40 feet.

FOCUS ON CAREERS



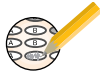
REAL LIFE CIVIL ENGINEERING

Civil engineers design and supervise the construction of roads, buildings, tunnels, bridges, and water supply systems.

CAREER LINK

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Test Preparation



TECHNOLOGY Using a square viewing screen on a graphing calculator, graph a line that passes through the origin and has a slope of 1.

53. Write an equation of the line you graphed. Approximately what angle does the line form with the x -axis?
54. Graph a line that passes through the origin and has a slope of 2. Write an equation of the line. When you doubled the slope, did the measure of the angle formed with the x -axis double?
55. **MULTIPLE CHOICE** If two different lines with equations $y = m_1x + b_1$ and $y = m_2x + b_2$ are parallel, which of the following must be true?
- (A) $b_1 = b_2$ and $m_1 \neq m_2$ (B) $b_1 \neq b_2$ and $m_1 \neq m_2$
 (C) $b_1 \neq b_2$ and $m_1 = m_2$ (D) $b_1 = b_2$ and $m_1 = m_2$
 (E) None of these

56. **MULTIPLE CHOICE** Which of the following is an equation of a line parallel to $y - 4 = -\frac{1}{2}x$?
- (A) $y = \frac{1}{2}x - 6$ (B) $y = 2x + 1$ (C) $y = -2x + 3$
 (D) $y = \frac{7}{2}x - 1$ (E) $y = -\frac{1}{2}x - 8$

★ Challenge

57. **xy USING ALGEBRA** Find a value for k so that the line through $(4, k)$ and $(-2, -1)$ is parallel to $y = -2x + \frac{3}{2}$.
58. **xy USING ALGEBRA** Find a value for k so that the line through $(k, -10)$ and $(5, -6)$ is parallel to $y = -\frac{1}{4}x + 3$.

MIXED REVIEW

RECIPROALS Find the reciprocal of the number. (Skills Review, p. 788)

59. 20 60. -3 61. -11 62. 340
 63. $\frac{3}{7}$ 64. $-\frac{13}{3}$ 65. $-\frac{1}{2}$ 66. 0.25

MULTIPLYING NUMBERS Evaluate the expression. (Skills Review, p. 785)

67. $\frac{3}{4} \cdot (-12)$ 68. $-\frac{3}{2} \cdot \left(-\frac{8}{3}\right)$ 69. $-10 \cdot \frac{7}{6}$ 70. $-\frac{2}{9} \cdot (-33)$

PROVING LINES PARALLEL Can you prove that lines m and n are parallel? If so, state the postulate or theorem you would use. (Review 3.4)

