

1.7

Introduction to Perimeter, Circumference, and Area

What you should learn

GOAL 1 Find the perimeter and area of common plane figures.

GOAL 2 Use a general problem-solving plan.

Why you should learn it

▼ To solve **real-life** problems about perimeter and area, such as finding the number of bags of seed you need for a field in **Example 4**.

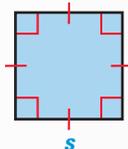
GOAL 1 REVIEWING PERIMETER, CIRCUMFERENCE, AND AREA

In this lesson, you will review some common formulas for perimeter, circumference, and area. You will learn more about area in Chapters 6, 11, and 12.

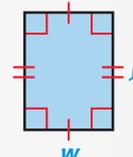
PERIMETER, CIRCUMFERENCE, AND AREA FORMULAS

Formulas for the perimeter P , area A , and circumference C of some common plane figures are given below.

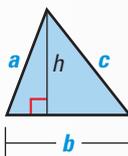
SQUARE
side length s
 $P = 4s$
 $A = s^2$



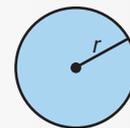
RECTANGLE
length ℓ and width w
 $P = 2\ell + 2w$
 $A = \ell w$



TRIANGLE
side lengths a , b , and c , base b , and height h
 $P = a + b + c$
 $A = \frac{1}{2}bh$



CIRCLE
radius r
 $C = 2\pi r$
 $A = \pi r^2$



Pi (π) is the ratio of the circle's circumference to its diameter.

The measurements of perimeter and circumference use units such as centimeters, meters, kilometers, inches, feet, yards, and miles. The measurements of area use units such as square centimeters (cm^2), square meters (m^2), and so on.

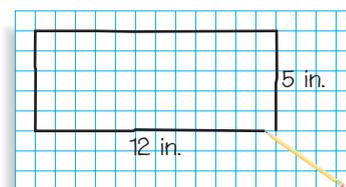
EXAMPLE 1 Finding the Perimeter and Area of a Rectangle

Find the perimeter and area of a rectangle of length 12 inches and width 5 inches.

SOLUTION

Begin by drawing a diagram and labeling the length and width. Then, use the formulas for perimeter and area of a rectangle.

$$\begin{aligned} P &= 2\ell + 2w & A &= \ell w \\ &= 2(12) + 2(5) & &= (12)(5) \\ &= 34 & &= 60 \end{aligned}$$



► So, the perimeter is 34 inches and the area is 60 square inches.

EXAMPLE 2 Finding the Area and Circumference of a Circle

Find the diameter, radius, circumference, and area of the circle shown at the right. Use 3.14 as an approximation for π .

STUDENT HELP

Study Tip

Some approximations for $\pi = 3.141592654 \dots$ are 3.14 and $\frac{22}{7}$.

SOLUTION

From the diagram, you can see that the diameter of the circle is

$$d = 13 - 5 = 8 \text{ cm.}$$

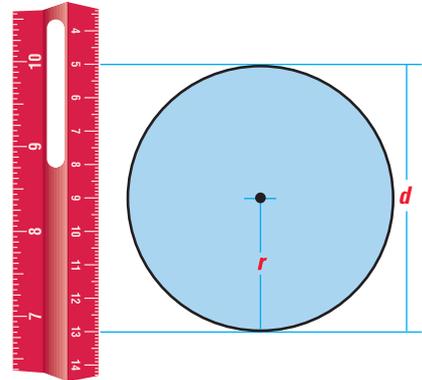
The radius is one half the diameter.

$$r = \frac{1}{2}(8) = 4 \text{ cm}$$

Using the formulas for circumference and area, you have

$$C = 2\pi r \approx 2(3.14)(4) \approx 25.1 \text{ cm}$$

$$A = \pi r^2 \approx 3.14(4^2) \approx 50.2 \text{ cm}^2.$$



EXAMPLE 3 Finding Measurements of a Triangle in a Coordinate Plane

Find the area and perimeter of the triangle defined by $D(1, 3)$, $E(8, 3)$, and $F(4, 7)$.

SOLUTION

Plot the points in a coordinate plane. Draw the height from F to side \overline{DE} . Label the point where the height meets \overline{DE} as G . Point G has coordinates $(4, 3)$.

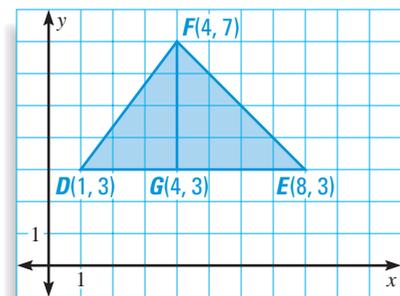
$$\text{base: } DE = 8 - 1 = 7$$

$$\text{height: } FG = 7 - 3 = 4$$

$$A = \frac{1}{2}(\text{base})(\text{height})$$

$$= \frac{1}{2}(7)(4)$$

$$= 14 \text{ square units}$$



To find the perimeter, use the Distance Formula.

$$EF = \sqrt{(4 - 8)^2 + (7 - 3)^2}$$

$$= \sqrt{(-4)^2 + 4^2}$$

$$= \sqrt{32}$$

$$= 4\sqrt{2} \text{ units}$$

$$DF = \sqrt{(4 - 1)^2 + (7 - 3)^2}$$

$$= \sqrt{3^2 + 4^2}$$

$$= \sqrt{25}$$

$$= 5 \text{ units}$$

► So, the perimeter is $DE + EF + DF = (7 + 4\sqrt{2} + 5)$, or $12 + 4\sqrt{2}$, units.

STUDENT HELP

Skills Review

For help with simplifying radicals, see page 799.

GOAL 2 USING A PROBLEM-SOLVING PLAN

A problem-solving plan can help you organize solutions to geometry problems.

A PROBLEM-SOLVING PLAN

1. Ask yourself what you need to solve the problem. Write a **verbal model** or **draw a sketch** that will help you find what you need to know.
2. **Label known and unknown facts** on or near your sketch.
3. Use labels and facts to **choose related definitions, theorems, formulas**, or other results you may need.
4. **Reason logically** to link the facts, using a proof or other written argument.
5. Write a **conclusion** that answers the original problem. **Check** that your reasoning is correct.

EXAMPLE 4 Using the Area of a Rectangle



SOCCER FIELD You have a part-time job at a school. You need to buy enough grass seed to cover the school's soccer field. The field is 50 yards wide and 100 yards long. The instructions on the seed bags say that one bag will cover 5000 square feet. How many bags do you need?

SOLUTION

Begin by rewriting the dimensions of the field in feet. Multiplying each of the dimensions by 3, you find that the field is 150 feet wide and 300 feet long.



VERBAL MODEL	$\text{Area of field} = \text{Bags of seed} \cdot \text{Coverage per bag}$
↓	
LABELS	<p>Area of field = $150 \cdot 300$ (square feet)</p> <p>Bags of seed = n (bags)</p> <p>Coverage per bag = 5000 (square feet per bag)</p>
↓	
REASONING	<p>$150 \cdot 300 = n \cdot 5000$ Write model for area of field.</p> <p>$\frac{150 \cdot 300}{5000} = n$ Divide each side by 5000.</p> <p>$9 = n$ Simplify.</p>

▶ You need 9 bags of seed.

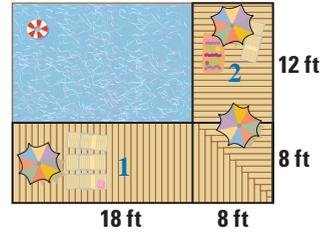
✓ **UNIT ANALYSIS** You can use *unit analysis* to verify the units of measure.

$$\text{ft}^2 = \text{bags} \cdot \frac{\text{ft}^2}{\text{bag}}$$

EXAMPLE 5 *Using the Area of a Square*



SWIMMING POOL You are planning a deck along two sides of a pool. The pool measures 18 feet by 12 feet. The deck is to be 8 feet wide. What is the area of the deck?



SOLUTION



DRAW A SKETCH
VERBAL MODEL
LABELS
REASONING

From your diagram, you can see that the area of the deck can be represented as the sum of the areas of two rectangles and a square.

$$\text{Area of deck} = \text{Area of rectangle 1} + \text{Area of rectangle 2} + \text{Area of square}$$

- Area of deck = A (square feet)
- Area of rectangle 1 = $8 \cdot 18$ (square feet)
- Area of rectangle 2 = $8 \cdot 12$ (square feet)
- Area of square = $8 \cdot 8$ (square feet)

$$A = 8 \cdot 18 + 8 \cdot 12 + 8 \cdot 8$$

Write model for deck area.

$$= 304$$

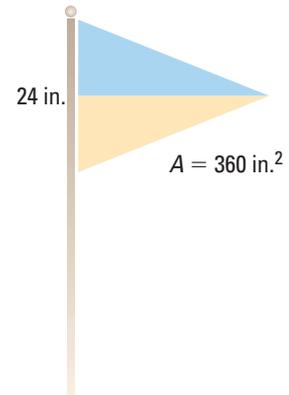
Simplify.

▶ The area of the deck is 304 square feet.

EXAMPLE 6 *Using the Area of a Triangle*



FLAG DESIGN You are making a triangular flag with a base of 24 inches and an area of 360 square inches. How long should it be?



SOLUTION



VERBAL MODEL
LABELS
REASONING

$$\text{Area of flag} = \frac{1}{2} \cdot \text{Base of flag} \cdot \text{Length of flag}$$

- Area of flag = 360 (square inches)
- Base of flag = 24 (inches)
- Length of flag = L (inches)

$$360 = \frac{1}{2}(24)L$$

Write model for flag area.

$$360 = 12L$$

Simplify.

$$30 = L$$

Divide each side by 12.

▶ The flag should be 30 inches long.

GUIDED PRACTICE

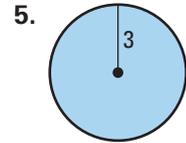
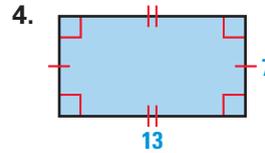
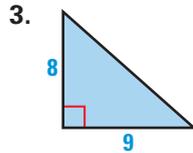
Vocabulary Check ✓

Concept Check ✓

Skill Check ✓

- The perimeter of a circle is called its ____? ____.
- Explain how to find the perimeter of a rectangle.

In Exercises 3–5, find the area of the figure. (Where necessary, use $\pi \approx 3.14$.)



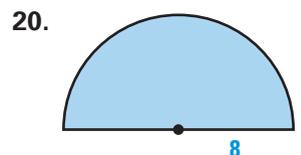
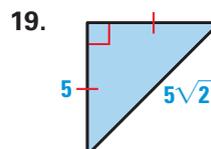
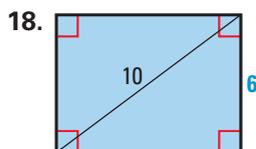
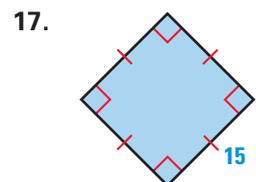
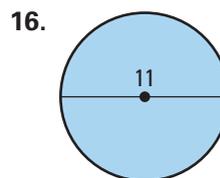
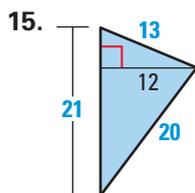
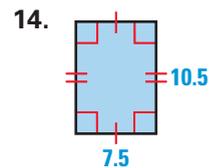
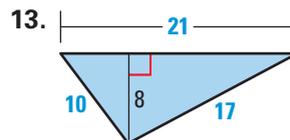
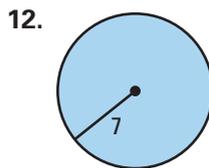
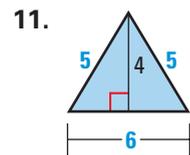
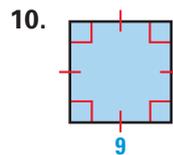
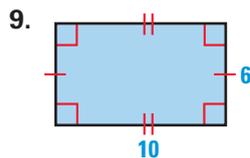
- The perimeter of a square is 12 meters. What is the length of a side of the square?
- The radius of a circle is 4 inches. What is the circumference of the circle? (Use $\pi \approx 3.14$.)
- FENCING** You are putting a fence around a rectangular garden with length 15 feet and width 8 feet. What is the length of the fence that you will need?

PRACTICE AND APPLICATIONS

STUDENT HELP

Extra Practice to help you master skills is on p. 804.

FINDING PERIMETER, CIRCUMFERENCE, AND AREA Find the perimeter (or circumference) and area of the figure. (Where necessary, use $\pi \approx 3.14$.)



STUDENT HELP

HOMEWORK HELP

- Example 1: Exs. 9–26
- Example 2: Exs. 9–26
- Example 3: Exs. 27–33
- Example 4: Exs. 34–40
- Example 5: Exs. 34–40
- Example 6: Exs. 41–48

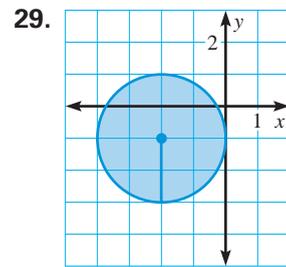
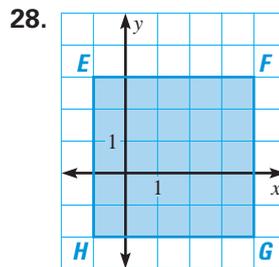
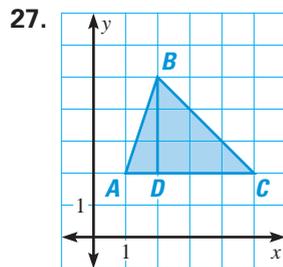
STUDENT HELP

INTERNET
HOMEWORK HELP
 Visit our Web site
www.mcdougallittell.com
 for help with problem
 solving in Exs. 21–26.

FINDING AREA Find the area of the figure described.

21. Triangle with height 6 cm and base 5 cm
22. Rectangle with length 12 yd and width 9 yd
23. Square with side length 8 ft
24. Circle with radius 10 m (Use $\pi \approx 3.14$.)
25. Square with perimeter 24 m
26. Circle with diameter 100 ft (Use $\pi \approx 3.14$.)

FINDING AREA Find the area of the figure.



FINDING AREA Draw the figure in a coordinate plane and find its area.

30. Triangle defined by $A(3, 4)$, $B(7, 4)$, and $C(5, 7)$
31. Triangle defined by $R(-2, -3)$, $S(6, -3)$, and $T(5, 4)$
32. Rectangle defined by $L(-2, -4)$, $M(-2, 1)$, $N(7, 1)$, and $P(7, -4)$
33. Square defined by $W(5, 0)$, $X(0, 5)$, $Y(-5, 0)$, and $Z(0, -5)$
34. **CARPETING** How many square yards of carpet are needed to carpet a room that is 15 feet by 25 feet?

35. **WINDOWS** A rectangular pane of glass measuring 12 inches by 18 inches is surrounded by a wooden frame that is 2 inches wide. What is the area of the window, including the frame?
36. **MILLENNIUM DOME** The largest fabric dome in the world, the Millennium Dome covers a circular plot of land with a diameter of 320 meters. What is the circumference of the covered land? What is its area? (Use $\pi \approx 3.14$.)

37. **SPREADSHEET** Use a spreadsheet to show many different possible values of length and width for a rectangle with an area of 100 m^2 . For each possible rectangle, calculate the perimeter. What are the dimensions of the rectangle with the smallest perimeter?

Perimeter of Rectangle								
	A	B	C	D	E	F	G	H
1	Length	1.00	2.00	3.00	4.00	5.00	6.00	...
2	Width	100.00	50.00	33.33	25.00	20.00	16.67	...
3	Area	100.00	100.00	100.00	100.00	100.00	100.00	...
4	Perimeter	202.00	104.00	72.67	58.00	50.00	45.33	...
5								

FOCUS ON APPLICATIONS



REAL LIFE
MILLENNIUM DOME
 Built for the year 2000, this dome in Greenwich, England, is over 50 m tall and is covered by more than 100,000 square meters of fabric.

INTERNET
APPLICATION LINK
www.mcdougallittell.com

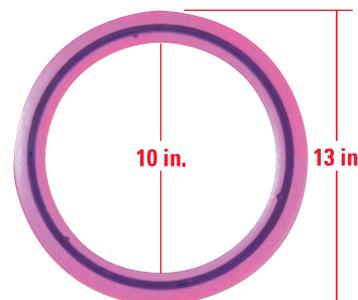
FOCUS ON APPLICATIONS



CRANBERRIES

Cranberries were once called “bounceberries” because they bounce when they are ripe.

38. **CRANBERRY HARVEST** To harvest cranberries, the field is flooded so that the berries float. The berries are gathered with an inflatable boom. What area of cranberries can be gathered into a circular region with a radius of 5.5 meters? (Use $\pi \approx 3.14$.)
39. **BICYCLES** How many times does a bicycle tire that has a radius of 21 inches rotate when it travels 420 inches? (Use $\pi \approx 3.14$.)
40. **FLYING DISC** A plastic flying disc is circular and has a circular hole in the middle. If the diameter of the outer edge of the ring is 13 inches and the diameter of the inner edge of the ring is 10 inches, what is the area of plastic in the ring? (Use $\pi \approx 3.14$.)



LOGICAL REASONING Use the given measurements to find the unknown measurement. (Where necessary, use $\pi \approx 3.14$.)

41. A rectangle has an area of 36 in.^2 and a length of 9 in. Find its perimeter.
42. A square has an area of $10,000 \text{ m}^2$. Find its perimeter.
43. A triangle has an area of 48 ft^2 and a base of 16 ft. Find its height.
44. A triangle has an area of 52 yd^2 and a height of 13 yd. Find its base.
45. A circle has an area of $200\pi \text{ cm}^2$. Find its radius.
46. A circle has an area of 1 m^2 . Find its diameter.
47. A circle has a circumference of 100 yd. Find its area.
48. A right triangle has sides of length 4.5 cm, 6 cm, and 7.5 cm. Find its area.

MULTI-STEP PROBLEM Use the following information.

Earth has a radius of about 3960 miles at the equator. Because there are 5280 feet in one mile, the radius of Earth is about 20,908,800 feet.

- Suppose you could wrap a cable around Earth to form a circle that is snug against the ground. Find the length of the cable in feet by finding the circumference of Earth. (Assume that Earth is perfectly round. Use $\pi \approx 3.14$.)
- Suppose you add 6 feet to the cable length in part (a). Use this length as the circumference of a new circle. Find the radius of the larger circle.
- Use your results from parts (a) and (b) to find how high off of the ground the longer cable would be if it was evenly spaced around Earth.
- Would the answer to part (c) be different on a planet with a different radius? Explain.

Test Preparation



Challenge

50. **DOUBLING A RECTANGLE'S SIDES** The length and width of a rectangle are doubled. How do the perimeter and area of the new rectangle compare with the perimeter and area of the original rectangle? Illustrate your answer.

MIXED REVIEW

SKETCHING FIGURES Sketch the points, lines, segments, and rays.

(Review 1.2 for 2.1)

51. Draw opposite rays using the points A , B , and C , with B as the initial point for both rays.
52. Draw four noncollinear points, W , X , Y , and Z , no three of which are collinear. Then sketch \overleftrightarrow{XY} , \overleftrightarrow{YW} , \overleftrightarrow{XZ} and \overleftrightarrow{ZY} .

 **USING ALGEBRA** Plot the points in a coordinate plane and sketch $\angle DEF$. Classify the angle. Write the coordinates of one point in the interior of the angle and one point in the exterior of the angle. (Review 1.4)

- | | | | |
|--------------------------------------------|-------------------------------------------|-----------------------------------------|--------------------------------------------|
| 53. $D(2, -2)$
$E(4, -3)$
$F(6, -2)$ | 54. $D(0, 0)$
$E(-3, 0)$
$F(0, -2)$ | 55. $D(0, 1)$
$E(2, 3)$
$F(4, 1)$ | 56. $D(-3, -2)$
$E(3, -4)$
$F(1, 3)$ |
|--------------------------------------------|-------------------------------------------|-----------------------------------------|--------------------------------------------|

FINDING THE MIDPOINT Find the coordinates of the midpoint of a segment with the given endpoints. (Review 1.5)

- | | |
|------------------------------|------------------------------|
| 57. $A(0, 0)$, $B(5, 3)$ | 58. $C(2, -3)$, $D(4, 4)$ |
| 59. $E(-3, 4)$, $F(-2, -1)$ | 60. $G(-2, 0)$, $H(-7, -6)$ |
| 61. $J(0, 5)$, $K(14, 1)$ | 62. $M(-44, 9)$, $N(6, -7)$ |

QUIZ 3

Self-Test for Lessons 1.6 and 1.7

In Exercises 1–4, find the measure of the angle. (Lesson 1.6)

- | | |
|---------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------------------------------------------------------|
| 1. Complement of $\angle A$; $m\angle A = 41^\circ$ | 2. Supplement of $\angle B$; $m\angle B = 127^\circ$ |
| 3. Supplement of $\angle C$; $m\angle C = 22^\circ$ | 4. Complement of $\angle D$; $m\angle D = 35^\circ$ |
| 5. $\angle A$ and $\angle B$ are complementary. The measure of $\angle A$ is five times the measure of $\angle B$. Find $m\angle A$ and $m\angle B$. (Lesson 1.6) | |

In Exercises 6–9, use the given information to find the unknown measurement. (Lesson 1.7)

6. Find the area and circumference of a circle with a radius of 18 meters. (Use $\pi \approx 3.14$.)
7. Find the area of a triangle with a base of 13 inches and a height of 11 inches.
8. Find the area and perimeter of a rectangle with a length of 10 centimeters and a width of 4.6 centimeters.
9. Find the area of a triangle defined by $P(-3, 4)$, $Q(7, 4)$, and $R(-1, 12)$.
10.  **WALLPAPER** You are buying rolls of wallpaper to paper the walls of a rectangular room. The room measures 12 feet by 24 feet and the walls are 8 feet high. A roll of wallpaper contains 28 ft^2 . About how many rolls of wallpaper will you need? (Lesson 1.7)